# European Gas Market Model

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## Indices

- Seasons: $s = 1, ..., S(= 12)$ months, most likely aligned with the gas storage year.
- Markets: $m = 1, ..., M$. Third countries (e.g. Russia) are not treated as markets.
- Producers: $p = 1, ..., P$. Local production only.
- Pipelines and LNG connections: $f = 1, ..., F$. Each direction has a separate index.
- Storages: $g = 1, ..., G$.
- Take-or-pay forward contracts: $c = 1, ..., C$. Import from third countries or transit through the region.
- Flow-limit combinations: $k = 1, ..., K$.

## "Pointers"

$\Pi_{mp} = 1$ if producer $p$ is in market $m$, 0 otherwise. $\Gamma_{mg} = 1$ if storage $g$ is in market $m$, 0 otherwise. $\Phi_{mf} = 1$ if the direction of connection $f$ is into market $m$, -1 if it is out of market $m$, and 0 otherwise. $\Psi_{kf}$ is the weight of the flow on connection $f$ in flow-limit combination $k$ (typically either 0 or 1). $\Upsilon_k = 0$ if the flow-limit concerns the physical flow, and $\Upsilon_k = 1$ if the limit is on spot-traded quantities only. $0 \leq \Omega^s_{fc} \leq 1$ is the fraction of the TOP contract $c$ that flows through connection $f$ in season $s$ (exogenous input parameter, sums to 1 across parallel routes). $\Lambda_{cm}$ is the weight with which the spot price of market $m$ is used as a reference value in pricing long-term contract $c$.

## Decision variables and associated constraints

- Production: $e^s_p$ $[S \times P]$
  - Production capacity: $ke^s_p$, $Ke^s_p$.
- Spot trade in the default direction: $t^s_f$ $[S \times F]$
  - Spot trade restriction (if any): $Kt^s_f$.
- Backhaul trade: $b^s_f$ $[S \times F]$
  - Backhaul capacity: $Kb^s_f$.
- Forward delivery: $d^s_c$ $[S \times C]$ (up to the TOP monthly minimum), $D^s_c$ $[S \times C]$ (from the TOP monthly minimum to the TOP monthly maximum)
  - TOP monthly min/max: $Kd^s_c$, $KD^s_c$. Upper limit on "cheap" and costly TOP deliveries.
- Injection: $i^s_g$ $[S \times G]$
  - Injection capacity: $Ki^s_g$
- Withdrawal: $w^s_g$ $[S \times G]$
  - Withdrawal capacity: $Kw^s_g$

## Parameters

- Production costs: $ce_p^s \leq Ce_p^s$. Marginal costs at zero production and the maximum production level. If equality holds, marginal production costs are constant, otherwise they are linearly increasing from $ce_p^s$ to $Ce_p^s$.
- Combined entry-exit fees in the default flow direction: $Ct_f^s$. Applies to forward contracts, too, not just spot trade.
- Combined entry-exit fees for backhaul deliveries: $Cb_f^s$.
- Spot import/export prices: $Pt_f^s$. The price (cost) paid for spot imports, or received for exports. Counts with third countries only, zero on all internal borders! E.g. $Pt_{RU \to UA} > 0$ (import cost), $Pt_{SK \to AT} = 0$, $Pt_{GR \to TR} < 0$ (export benefit).
- Long-term import price constants: $Md_c^s$, $MD_c^s$
- Injection charges: $Ci_g^s$
- Withdrawal charges: $Cw_g^s$
- Maximum yearly production level (might be less than the sum of the months): $Ke_p$.
- Physical flow capacity: $kx_f^s$, $Kx_f^s$. Minimum and maximum pipeline flow.
- Flow-limit combination upper constraints: $Kc_k^s$.
- Maximum backhaul ratio: $Rb_f^s$. Maximum fraction of contractual deliveries on a pipeline that can be exploited by virtual reverse flow.
- Working gas capacity: $Kg_g$
- Gas inventories: $0 \leq I_g^0, I_g^S \leq Kg_g$ (starting and finishing)
- TOP yearly min/max: $kd_c$, $KD_c$. Lower and upper limit on the sum of all forward deliveries.

## Derived variables

### Physical flow

$$x_f^s = t_f^s - b_f^s + \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s)$$

### Consumption

$$\begin{aligned} Q_m^s &= \sum_p \Pi_{mp} \cdot e_p^s + \sum_f \Phi_{mf} \cdot x_f^s + \sum_g \Gamma_{mg} \cdot \left(w_g^s - i_g^s\right) \\ &= \sum_p \Pi_{mp} \cdot e_p^s + \sum_f \Phi_{mf} \cdot \left[t_f^s - b_f^s + \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s)\right] + \sum_g \Gamma_{mg} \cdot \left(w_g^s - i_g^s\right) \end{aligned}$$

### Prices

$$P_m^s\left(Q_m^s\right) = A_m^s - B_m^s \cdot Q_m^s$$

(linear inverse demand function).

### Production costs (linear marginal costs)

$$C_p^s\left(e_p^s\right) = \frac{Ce_p^s - ce_p^s}{2 \cdot Ke_p^s} \cdot \left(e_p^s\right)^2 + ce_p^s \cdot e_p^s$$

**Long-term import prices**

$$Pd_c^s = Md_c^s + \sum_m \Lambda_{cm} \cdot P_m^s \left(Q_m^s\right)$$

$$PD_c^s = MD_c^s + \sum_m \Lambda_{cm} \cdot P_m^s \left(Q_m^s\right)$$

## Objective function

Welfare is given by gross consumer surplus minus the costs (and benefits) of production, storage and trade (including imports and exports with outside countries, which is where the benefits may arise):

$$\begin{aligned} W = \; & \sum_s \beta^s \left\{ \sum_m \left[ \int_0^{Q_m^s} P_m^s(Q)\, dQ \right] - \sum_p C_p^s\left(e_p^s\right) - \sum_g Ci_g^s \cdot i_g^s - \sum_g Cw_g^s \cdot w_g^s \right. \\ & \left. - \sum_f Ct_f^s \cdot \left[ \sum_c \Omega_{fc}^s \cdot \left(d_c^s + D_c^s\right) + t_f^s \right] - \sum_f Cb_f^s \cdot b_f^s - \sum_f Pt_f^s \cdot \left(t_f^s - b_f^s\right) - \sum_c Pd_c^s \cdot d_c^s - \sum_c PD_c^s \cdot D_c^s \right\} \end{aligned}$$

$\beta$ denotes the monthly discount factor.[1] Substituting the derived variables (other than market quantities):

$$\begin{aligned} W = \; & \sum_s \beta^s \left\{ \sum_m \left[ A_m^s \cdot Q_m^s - \frac{B_m^s}{2} \cdot \left(Q_m^s\right)^2 \right] - \sum_p \left[ \frac{Ce_p^s - ce_p^s}{2 \cdot Ke_p^s} \cdot \left(e_p^s\right)^2 + ce_p^s \cdot e_p^s \right] - \sum_g Ci_g^s \cdot i_g^s - \sum_g Cw_g^s \cdot w_g^s \right. \\ & - \sum_f Ct_f^s \cdot \left[ \sum_c \Omega_{fc}^s \cdot \left(d_c^s + D_c^s\right) + t_f^s \right] - \sum_f Cb_f^s \cdot b_f^s - \sum_f Pt_f^s \cdot \left(t_f^s - b_f^s\right) \\ & \left. - \sum_c \left[ Md_c^s + \sum_m \Lambda_{cm} \cdot P_m^s\left(Q_m^s\right) \right] \cdot d_c^s - \sum_c \left[ MD_c^s + \sum_m \Lambda_{cm} \cdot P_m^s\left(Q_m^s\right) \right] \cdot D_c^s \right\} \end{aligned}$$

## Constraints (other than on the decision variables)

### Production

Upper yearly production limit:

$$\sum_s e_p^s \leq Ke_p$$

### Transportation

Lower pipeline capacity:

$$t_f^s - b_f^s + \sum_c \Omega_{fc}^s \cdot \left(d_c^s + D_c^s\right) \geq kx_f^s \quad \forall s$$

Upper pipeline capacity:

$$t_f^s - b_f^s + \sum_c \Omega_{fc}^s \cdot \left(d_c^s + D_c^s\right) \leq Kx_f^s \quad \forall s$$

Combined flow-limits:

$$\sum_f \Psi_{kf} \cdot \left[t_f^s - b_f^s\right] + \left(1 - \Upsilon_k\right) \cdot \sum_f \Psi_{kf} \cdot \left[ \sum_c \Omega_{fc}^s \cdot \left(d_c^s + D_c^s\right) \right] \leq Kc_k^s \quad \forall s$$

Maximum virtual reverse flow as a fraction of contractual deliveries:

$$Rb_f^s \cdot \sum_c \Omega_{fc}^s \cdot \left(d_c^s + D_c^s\right) - b_f^s \geq 0 \quad \forall s$$

### Contracts

TOP yearly limits:

$$\sum_s d_c^s + \sum_s D_c^s \geq kd_c$$

$$\sum_s d_c^s + \sum_s D_c^s \leq KD_c$$

[1] Therefore $\beta^{12} = \frac{1}{1+r}$, where $r$ is the yearly interest rate used for discounting.

**Storages**

No negative storage in any month:

$$\sum_{1}^{\overline{s}} w_g^s - \sum_{1}^{\overline{s}} i_g^s \leq I_g^0 \quad \forall \overline{s} \in \{1, ..., S\}$$

No overloaded storage in any month:

$$\sum_{1}^{\overline{s}} i_g^s - \sum_{1}^{\overline{s}} w_g^s \leq Kg_g - I_g^0 \quad \forall \overline{s} \in \{1, ..., S\}$$

Storage reloading to pre-specified level by the end of the year:

$$\sum_{s} i_g^s - \sum_{s} w_g^s \geq I_g^S - I_g^0$$

## The Langrangian

$$\begin{array}{rl} L = & \sum_s \beta^s \left\{ \sum_m \left[ A_m^s \cdot Q_m^s - \frac{B_m^s}{2} \cdot (Q_m^s)^2 \right] - \sum_p \left[ \frac{Ce_p^s - ce_p^s}{2 \cdot Ke_p^s} \cdot \left(e_p^s\right)^2 + ce_p^s \cdot e_p^s \right] - \sum_g Ci_g^s \cdot i_g^s - \sum_g Cw_g^s \cdot w_g^s \right. \\ & - \sum_f Ct_f^s \cdot \left[ \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s) + t_f^s \right] - \sum_f Cb_f^s \cdot b_f^s - \sum_f Pt_f^s \cdot \left( t_f^s - b_f^s \right) \\ & \left. - \sum_c \left[ Md_c^s + \sum_m \Lambda_{cm} \cdot P_m^s \left(Q_m^s\right) \right] \cdot d_c^s - \sum_c \left[ MD_c^s + \sum_m \Lambda_{cm} \cdot P_m^s \left(Q_m^s\right) \right] \cdot D_c^s \right\} \\ + & \sum_p \varepsilon_p \left( Ke_p - \sum_s e_p^s \right) \\ + & \sum_s \sum_f \phi_f^s \left( t_f^s - b_f^s + \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s) - kx_f^s \right) \\ + & \sum_s \sum_f \varphi_f^s \left\{ Kx_f^s - t_f^s + b_f^s - \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s) \right\} \\ + & \sum_s \sum_k \kappa_k^s \left\{ Kc_k^s - \sum_f \Psi_{kf} \cdot \left( t_f^s - b_f^s \right) - (1 - \Upsilon_k) \cdot \sum_f \Psi_{kf} \cdot \left[ \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s) \right] \right\} \\ + & \sum_c \tau_c \left( \sum_s d_c^s + \sum_s D_c^s - kd_c \right) \\ + & \sum_c \theta_c \left( KD_c - \sum_s d_c^s - \sum_s D_c^s \right) \\ + & \sum_s \sum_f \rho_f^s \left[ Rb_f^s \cdot \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s) - b_f^s \right] \\ + & \sum_g \sum_{\overline{s}=1}^{S} \mu_g^{\overline{s}} \left( I_g^0 + \sum_1^{\overline{s}} i_g^s - \sum_1^{\overline{s}} w_g^s \right) \\ + & \sum_g \sum_{\overline{s}=1}^{S} \nu_g^{\overline{s}} \left( Kg_g - I_g^0 - \sum_1^{\overline{s}} i_g^s + \sum_1^{\overline{s}} w_g^s \right) \\ + & \sum_g \lambda_g \left( I_g^0 - I_g^S + \sum_s i_g^s - \sum_s w_g^s \right) \end{array}$$

Lower and upper constraints on the decision variables are not explicitly included in the Lagrangian, but are taken into account in the MLCP solution algorithm.

We make one small, but important modification to the welfare maximization problem. In the import pricing formulas, the part

$$\sum_m \Lambda_{cm} \cdot P_m^s \left(Q_m^s\right)$$

captures the dependence of import contract costs on local equilibrium prices (e.g. Dutch TTF price). We will assume that all market participants take the price $P_m^s\left(Q_m^s\right)$ as exogenous, and do not recognize that their own actions on the local market may influence the cost of long-term imports. Thus, the first derivative of$P_m^s\left(Q_m^s\right)$ with respect to any decision variable will be zero in the import pricing formulas.

## First derivatives of the Lagrangian with respect to decision variables

$$\frac{\partial L}{\partial e_p^s} = \beta^s \left[ \sum_m \Pi_{mp} \cdot A_m^s - \sum_m \Pi_{mp} \cdot B_m^s \cdot Q_m^s - \frac{Ce_p^s - ce_p^s}{Ke_p^s} \cdot e_p^s - ce_p^s \right] - \varepsilon_p$$

$$\frac{\partial L}{\partial t_f^s} = \beta^s \left[ \sum_m \Phi_{mf} \cdot A_m^s - \sum_m \Phi_{mf} \cdot B_m^s \cdot Q_m^s - Ct_f^s - Pt_f^s \right] + \phi_f^s - \varphi_f^s - \sum_k \Psi_{kf} \cdot \kappa_k^s$$

$$\frac{\partial L}{\partial b_f^s} = \beta^s \left[ - \sum_m \Phi_{mf} \cdot A_m^s + \sum_m \Phi_{mf} \cdot B_m^s \cdot Q_m^s - Cb_f^s + Pt_f^s \right] - \phi_f^s + \varphi_f^s + \sum_k \Psi_{kf} \cdot \kappa_k^s - \rho_f^s$$

$$
\begin{aligned}
\frac{\partial L}{\partial d_c^s} &= \beta^s \Big[\textstyle\sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot A_m^s - \sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot B_m^s \cdot Q_m^s - \sum_f Ct_f^s \cdot \Omega_{fc}^s \\
&\quad -Md_c^s - \textstyle\sum_m \Lambda_{cm} \cdot A_m^s + \sum_m \Lambda_{cm} \cdot B_m^s \cdot Q_m^s\Big] \\
&\quad + \textstyle\sum_f \phi_f^s \cdot \Omega_{fc}^s - \sum_f \varphi_f^s \cdot \Omega_{fc}^s - \sum_k (1-\Upsilon_k) \cdot \kappa_k^s \cdot \left(\sum_f \Psi_{kf} \cdot \Omega_{fc}^s\right) + \tau_c - \theta_c + \sum_f \rho_f^s \cdot Rb_f^s \cdot \Omega_{fc}^s
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial D_c^s} &= \beta^s \Big[\textstyle\sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot A_m^s - \sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot B_m^s \cdot Q_m^s - \sum_f Ct_f^s \cdot \Omega_{fc}^s \\
&\quad -MD_c^s - \textstyle\sum_m \Lambda_{cm} \cdot A_m^s + \sum_m \Lambda_{cm} \cdot B_m^s \cdot Q_m^s\Big] \\
&\quad + \textstyle\sum_f \phi_f^s \cdot \Omega_{fc}^s - \sum_f \varphi_f^s \cdot \Omega_{fc}^s - \sum_k (1-\Upsilon_k) \cdot \kappa_k^s \cdot \left(\sum_f \Psi_{kf} \cdot \Omega_{fc}^s\right) + \tau_c - \theta_c + \sum_f \rho_f^s \cdot Rb_f^s \cdot \Omega_{fc}^s
\end{aligned}
$$

$$
\frac{\partial L}{\partial i_g^s} = \beta^s \left[-\sum_m \Gamma_{mg} \cdot A_m^s + \sum_m \Gamma_{mg} \cdot B_m^s \cdot Q_m^s - Ci_g^s\right] + \sum_{\overline{s}=s}^{S} \mu_g^{\overline{s}} - \sum_{\overline{s}=s}^{S} \nu_g^{\overline{s}} + \lambda_g
$$

$$
\frac{\partial L}{\partial w_g^s} = \beta^s \left[\sum_m \Gamma_{mg} \cdot A_m^s - \sum_m \Gamma_{mg} \cdot B_m^s \cdot Q_m^s - Cw_g^s\right] - \sum_{\overline{s}=s}^{S} \mu_g^{\overline{s}} + \sum_{\overline{s}=s}^{S} \nu_g^{\overline{s}} - \lambda_g
$$

## Complementarity conditions in the MLCP

The first set of brackets on the left hand side contains linear combinations of the variables, while the second set contains constants (as in: $Ax + b \geq 0, x \geq 0, (\perp)$).

$$
\left\{\beta^s \cdot \sum_m \Pi_{mp} \cdot B_m^s \cdot Q_m^s + \beta^s \cdot \frac{Ce_p^s - ce_p^s}{Ke_p^s} \cdot e_p^s + \varepsilon_p\right\} + \left\{\beta^s \cdot ce_p^s - \beta^s \cdot \sum_m \Pi_{mp} \cdot A_m^s\right\} \geq 0 \quad e_p^s \geq 0 \quad (\perp)
$$

$$
\left\{\beta^s \cdot \sum_m \Phi_{mf} \cdot B_m^s \cdot Q_m^s - \phi_f^s + \varphi_f^s + \sum_k \Psi_{kf} \cdot \kappa_k^s\right\} + \left\{\beta^s \cdot Ct_f^s + \beta^s \cdot Pt_f^s - \beta^s \cdot \sum_m \Phi_{mf} \cdot A_m^s\right\} \geq 0 \quad t_f^s \geq 0 \quad (\perp)
$$

$$
\left\{-\beta^s \cdot \sum_m \Phi_{mf} \cdot B_m^s \cdot Q_m^s + \phi_f^s - \varphi_f^s - \sum_k \Psi_{kf} \cdot \kappa_k^s + \rho_f^s\right\} + \left\{\beta^s \cdot Cb_f^s - \beta^s \cdot Pt_f^s + \beta^s \cdot \sum_m \Phi_{mf} \cdot A_m^s\right\} \geq 0 \quad b_f^s \geq 0 \quad (\perp)
$$

$$
\begin{aligned}
&\Big\{\beta^s \cdot \textstyle\sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot B_m^s \cdot Q_m^s - \beta^s \cdot \sum_m \Lambda_{cm} \cdot B_m^s \cdot Q_m^s - \sum_f \phi_f^s \cdot \Omega_{fc}^s + \sum_f \varphi_f^s \cdot \Omega_{fc}^s \\
&\qquad + \textstyle\sum_k (1-\Upsilon_k) \cdot \kappa_k^s \cdot \left(\sum_f \Psi_{kf} \cdot \Omega_{fc}^s\right) - \tau_c + \theta_c - \sum_f \rho_f^s \cdot Rb_f^s \cdot \Omega_{fc}^s\Big\} \\
&\quad + \Big\{-\beta^s \cdot \textstyle\sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot A_m^s + \beta^s \cdot \sum_m \Lambda_{cm} \cdot A_m^s + \beta^s \cdot \sum_f Ct_f^s \cdot \Omega_{fc}^s + \beta^s \cdot Md_c^s\Big\} \geq 0 \\
&\qquad d_c^s \geq 0 \quad (\perp)
\end{aligned}
$$

$$
\begin{aligned}
&\Big\{\beta^s \cdot \textstyle\sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot B_m^s \cdot Q_m^s - \beta^s \cdot \sum_m \Lambda_{cm} \cdot B_m^s \cdot Q_m^s - \sum_f \phi_f^s \cdot \Omega_{fc}^s + \sum_f \varphi_f^s \cdot \Omega_{fc}^s \\
&\qquad + \textstyle\sum_k (1-\Upsilon_k) \cdot \kappa_k^s \cdot \left(\sum_f \Psi_{kf} \cdot \Omega_{fc}^s\right) - \tau_c + \theta_c - \sum_f \rho_f^s \cdot Rb_f^s \cdot \Omega_{fc}^s\Big\} \\
&\quad + \Big\{-\beta^s \cdot \textstyle\sum_m \left(\sum_f \Phi_{mf} \cdot \Omega_{fc}^s\right) \cdot A_m^s + \beta^s \cdot \sum_m \Lambda_{cm} \cdot A_m^s + \beta^s \cdot \sum_f Ct_f^s \cdot \Omega_{fc}^s + \beta^s \cdot MD_c^s\Big\} \geq 0 \\
&\qquad D_c^s \geq 0 \quad (\perp)
\end{aligned}
$$

$$
\left\{-\beta^s \cdot \sum_M \Gamma_{mg} \cdot B_m^s \cdot Q_m^s - \sum_{\overline{s}=s}^{S} \mu_g^{\overline{s}} + \sum_{\overline{s}=s}^{S} \nu_g^{\overline{s}} - \lambda_g\right\} + \left\{\beta^s \cdot Ci_g^s + \beta^s \cdot \sum_M \Gamma_{mg} \cdot A_m^s\right\} \geq 0 \quad i_g^s \geq 0 \quad (\perp)
$$

$$
\left\{\beta^s \cdot \sum_M \Gamma_{mg} \cdot B_m^s \cdot Q_m^s + \sum_{\overline{s}=s}^{S} \mu_g^{\overline{s}} - \sum_{\overline{s}=s}^{S} \nu_g^{\overline{s}} + \lambda_g\right\} + \left\{\beta^s \cdot Cw_g^s - \beta^s \cdot \sum_M \Gamma_{mg} \cdot A_m^s\right\} \geq 0 \quad w_g^s \geq 0 \quad (\perp)
$$

$$
\left\{-\sum_s e_p^s\right\} + \{Ke_p\} \geq 0 \quad \varepsilon_p \geq 0 \quad (\perp)
$$

$$
\left\{t_f^s - b_f^s + \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s)\right\} + \{-kx_f^s\} \geq 0 \quad \phi_f^s \geq 0 \quad (\perp)
$$

$$
\left\{-t_f^s + b_f^s - \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s)\right\} + \{Kx_f^s\} \geq 0 \quad \varphi_f^s \geq 0 \quad (\perp)
$$

$$\left\{-\sum_f \Psi_{kf} \cdot \left(t_f^s - b_f^s\right) - (1-\Upsilon_k) \cdot \sum_f \Psi_{kf} \cdot \left[\sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s)\right]\right\} + \{Kc_k^s\} \geq 0 \quad \kappa_k^s \geq 0 \quad (\perp)$$

$$\left\{\sum_s d_c^s + \sum_s D_c^s\right\} + \{-kd_c\} \geq 0 \quad \tau_c \geq 0 \quad (\perp)$$

$$\left\{-\sum_s d_c^s - \sum_s D_c^s\right\} + \{KD_c\} \geq 0 \quad \theta_c \geq 0 \quad (\perp)$$

$$\left\{Rb_f^s \cdot \sum_c \Omega_{fc}^s \cdot (d_c^s + D_c^s) - b_f^s\right\} \geq 0 \quad \rho_f^s \geq 0 \quad (\perp)$$

$$\left\{\sum_1^{\overline{s}} i_g^s - \sum_1^{\overline{s}} w_g^s\right\} + \left\{I_g^0\right\} \geq 0 \quad \mu_g^{\overline{s}} \geq 0 \quad (\perp)$$

$$\left\{-\sum_1^{\overline{s}} i_g^s + \sum_1^{\overline{s}} w_g^s\right\} + \left\{Kg_g - I_g^0\right\} \geq 0 \quad \nu_g^{\overline{s}} \geq 0 \quad (\perp)$$

$$\left\{\sum_s i_g^s - \sum_s w_g^s\right\} + \left\{I_g^0 - I_g^S\right\} \geq 0 \quad \lambda_g \geq 0 \quad (\perp)$$